Mathematical-Metacognitive Discourse as Aiding Comprehension of the Equality Concept in Mathematics among First Grade Students

Avivit Menachem

Submitted in partial fulfillment of the requirements for the Master's Degree in the School of Education, Bar-Ilan University
Abstract

In recent years, Israeli schools underwent changes in the field of mathematical education. Following the shifts in the field occurred in the United States and Europe, a higher emphasis has been placed on mathematical discourse, with focus on raising arguments and providing proof, explanations and justifications for mathematical actions in light of the constructive approach. Nowadays, the mathematical discourse is considered to be the primary factor in teaching and learning as studies indicate a tight bond between discourse, learning and reasoning. The axiom of the researchers is that the concept of discourse is identified absolutely with the concept of reasoning. Therefore, when a child speaks he tells us what he is thinking (Sfard & Kieran, 2001).

One of the main concepts in mathematics, crucial to developing algebraic reasoning among young children, is the concept of equality. Equality is a ratio expressing the idea that two mathematical expressions hold the same value (the full definition will follow later on). Several researchers (Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011) indicate that children in elementary classes relate to equality mark operatively, as a command to execute a mathematical operation, namely, to execute the calculation before it, as the number that follows it is perceived as the answer to the calculation. According to the researchers Falkner, Levi, & Carpenter (1999), children must understand that equality is a ration that expresses equivalence, meaning, two mathematical expressions holding the same value and concept. In order to establish such understanding, a concentrated effort of learning and assimilation is required. One of the more effective means for developing understanding and flexibility in regard with the concept of equality in children, rather than memorizing a sequence of rules to solve equations is a method of teaching that
relies on a meta-cognitive discourse which develops the students' learning and reasoning abilities. In recent decades, studies have shown the effectiveness of meta-cognitive methods of teaching in grooming the mathematical reasoning and discourse while strengthening the child's meta-cognition regarding the means of solution (Kramarski & Mevarech, 2003, Mevarech & Kramarski, 1997, 2014; Tzohar-Rozen & Kramarski, 2013).

The current study examines the effect and contribution of a mathematical – meta-cognitive discourse as aiding the success in the comprehension of the concept of equality in mathematics in terms of equivalence among first grade students. In addition, the study examines the various influences of support and meta-cognitive guidance in improving skills, such as the student's self-efficacy and level of preservation of the acquired knowledge. The concepts of self-efficacy and preservation of knowledge will be elaborated in the review chapter.

The study consists of three main questions:

- Will differences be found in the level of understanding of the concept of equality in terms of equivalence between the experiment group and the control group at the end of the study?
- Will differences be found between the study groups in the level of self-efficacy in dealing with solving a math problem on the concept of equality in terms of equivalence at the end of the study?
- Will differences be found between the study groups in the ability to preserve the acquired knowledge for a length of time regarding the concept of equality in terms of equivalence at the end of the study?

The sample consisted of 60 first grade students of a school in the center of Israel, randomly divided into two study groups: one class posed as an experimental group
whiles the other as a control group. The experimental group underwent early intervention (as elaborated in the method chapter) and a mathematics intervention program concerning the concept of equality in terms of equivalence, which included a meta-cognitive guidance based on leading questions according to the (IMPROVE) method. The control group underwent the same intervention program without emphasis on a structural meta-cognitive guidance.

In order to examine the effect of the program on educational achievements in mathematics, on the level of efficacy and on the ability to preserve the knowledge of the concept of equality in terms of equivalence over time, different evaluation tools, quantitative and qualitative, were transmitted to the students. The tools consisted of tests transmitted to each student in two time points: before and after the intervention. Furthermore, processes analysis including individual interviews and videotaped observations on the mathematical discourse in the study groups of both study groups, were conducted. The duration of the study was three months.

The study consists of two main stages: **in the first stage**, the mathematical knowledge regarding the concept of equality was tested in of all of the students. Following this, an intervention was conducted. The intervention included an early intervention on the experiment group only, in which the students prepared the aiding tools for the meta-cognitive guidance accompanying them throughout the study: a "think aloud" puzzle with which they experimented and played in various manners. Afterwards, both study groups underwent an intervention in order to learn the concept of equality in terms of equivalence, while the experiment group was given a structural meta-cognitive guidance accompanied by the aiding tool (the puzzle), whereas the control group was given the same intervention only without the structural meta-cognitive guidance. All of the lessons of the unit were
videotaped and transcribed. A qualitative analysis of the mathematical discourse was conducted on lesson number two, called "marksmanship". The transcripts of this discourse were divided into meaning sentences and analyzed according to the meta-cognitive model by Schraw (1998), in accordance with declarative, conditional and procedural knowledge, as well as the criteria created for the purpose of analyzing the discourse. Judges reliability for evaluating the discourse was conducted. In both study groups a study practice was conducted on the character of the mathematical discourse inside the group, mediated by the teacher, before the intervention. Following the intervention, a mathematical knowledge test (post) in the concept of equality was conducted on both groups.

At the end of the study, an assessment assignment of the student's self-efficacy and examination of strategic knowledge was conducted, in order to examine whether differences among both groups will be found in this variable.

The second stage of the research was the interview stage "thinking aloud". At the end of the study, a group of students underwent several interviews. In this stage participated 20 students who were randomly selected: 10 students from the experiment group and 10 students from the control group. At first, the students received a questionnaire with six exercises that they were required to solve. Afterwards, they were individually questioned on the details of the questionnaire and on the reason they chose a particular way of solving. This data also underwent a quantitative and qualitative analyzes (as elaborated in the method chapter).

Approximately two months of the end of the study, an assignment aimed at estimating the assessment of the ability to preserve the students' acquired
knowledge of the concept of equality in terms of equivalence, was conducted. This assignment underwent a quantitative analysis.

It was found that students who were exposed to structural meta-cognitive guidance, basing on the principles of IMPROVE, portrayed higher levels of mathematical discourse, used more efficient strategies for solving and had higher self-efficacy in dealing with an assignment in the concept of equality, than students who were not exposed to structural meta-cognitive guidance. Furthermore, it was found that the meta-cognitive mathematical discourse of the experiment group was more diverse, rich and filled with mathematical contents and meta-cognitive behavior than the control group. Additionally, it was found that the ability of preserving acquired knowledge was expressed more efficiently among the experiment group.

Meta-cognitive intervention programs at a young age are less frequent in the framework of learning in schools. Specifically, the study of the concept of equality according to the principles of the IMPROVE method, was not studied until today. This study provides theoretical contribution to expanding the understanding of the difficulties in grasping the concept of equality. Such contribution includes a methodological contribution, namely, in building criteria for analyzing a meta-cognitive mathematical discourse as well as a practical contribution realized in aiding the intervention manners and meta-cognitive teaching starting from preschoolers.