

**Effects of a Fading Model for Metacognitive Judgment  
in Mathematical Problem Solving among Students  
With and Without Mathematical Disability**

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## ABSTRACT

Solving word problems is considered one of the hardest subjects for students to comprehend, and is defined as an important standard of comprehension (NCTM, 2014). The PISA international studies (2012) and the TIMSS (2011) focus on solution of complex mathematical problems that aside from knowledge of necessary mathematical content, also require the learner to use a variety of cognitive skills (TIMSS, 2007, 2011) on ascending thought levels: mastery of facts, procedures and mathematical concepts; solving routine problems of the type pupils encounter in class; and solving non-routine problems that demand application of mathematical procedures in an unfamiliar or complex context. Furthermore, researchers emphasize the importance of developing high order thinking skills, such as transfer to subjects not yet studied and the capacity for long-term retention of knowledge (Jitendra, Griffin, Haria, Leh, Adams, & Kaduvetoor, 2007). These are difficult skills for most pupils, particularly for pupils with mathematical learning disabilities.

Pupils with mathematical learning disabilities have difficulty with various aspects of learning: mastery of basic facts, application of algorithms, and correct use of mathematical terminology. In addition, they find it hard to grasp the meaning of texts in word problems (Garnett, 1998). Studies have also found that mathematically impaired pupils' level of mastery of metacognitive tools matches the level of younger pupils (Desoete & Roeyers, 2005). Pupils with learning disabilities are generally included in regular education classes, where pupils with learning disabilities and pupils at risk for failure in mathematics study together with those without disabilities. Pupils at risk for mathematics failure are those that show reasonable achievement, but find it difficult to make accurate self-assessment of their performance (Labuhn, Zimmerman, & Hasselhorn, 2010).

The many difficulties involved in acquiring skills for solving numerical reasoning problems present a need for systematic instructional scaffolding in mathematics.

**The Fading model** is defined in the literature as a significant level in a hierarchy of supports provided to the learner. This model is based on gradually diminishing support as a stage in preparing learners for independent activity (Puntambekar & Hubscher, 2005). In most cases where the Fading model is utilized, the ranked activity is directed toward learning content (mathematics, sciences, electrical engineering)

rather than toward the thought process of problem solving in which the pupil regulates the solving process with the aid of his/her own questions, such as: **What** is the knowledge base required (verbal, mathematical, strategic); **Which** strategies ought to be selected; **When and why** should they be applied during the solution process; and finally, **How** should I think about the whole process? These questions relate to the essence of Self-Regulated Learning.

In Self-Regulated Learning (SRL), the learner ranks goals for himself, and similarly plans, supervises and assesses his work while relating to the cognitive, the meta-cognitive, and the motivational-emotional components (Pintrich, Wolters & Baxter, 2000). The extensive literature in the field of meta-cognitive regulation proves its considerable contribution both in the areas of content as well as skills for self-regulation among younger and older learners (Kramarski, Weiss, & Sharon, 2013). Meta-cognitive support by means of giving prompts to pupils is an efficient method to encourage active learning. One method of meta-cognitive support is providing prompts that act on the individual's meta-cognitive processes, by using self-questions (Schoenfeld, 1985).

No unequivocal answer has yet to be given as to the desired timing for providing prompts, in different stages of working on the solution: before, during or after. Support prior to solution is intended for intensive planning, whereas support given after solution is intended as supervision and reflection. It has been found that reflection is an important, efficient strategy during problem solving.

One of the characteristics of metacognition is metacognitive judgment, defined as the learner's developing ability to assess the correctness of his answer on the given task (Goldsmith & Koriati, 2008). The judgment process enables the learner to supervise the strategies, which he has used, to examine their effectiveness, and to regulate his performance at each stage of the solution: before, during or after its implementation.

The uniqueness of the current study is its combination of the different areas it examines: solution of numerical reasoning problems, self-regulated learning, designing scaffolding according to the Fading model, consideration of differences among learners in the class, and training for metacognitive judgment, while relating to younger pupils.

**The study has four goals:**

1. **Development** of four unique intervention programs. These are based on providing different metacognitive prompts for numerical reasoning problem solving, while fostering metacognitive judgment at different stages: before or after. In two of the intervention programs, prompts were given according to the Fading model and training metacognitive judgment, while relating to younger pupils.
2. **Investigation of** the influence of the Fading model on solving numerical reasoning problems and on self-regulation in learning during the stage prior to and the stage after problem solving, as compared to providing the same prompts on a fixed basis (without fading).
3. **Assessment** of each intervention program's effectiveness on the ability to solve numerical reasoning problems and on self-regulated learning. This was done using tools that focus on the learner's self-report (questionnaires) and examination of the learning process in real time (thinking aloud and self-judgment), as well as assessment of the program's effectiveness in retention of learned material and the ability to transfer over time.
4. Evaluation of each intervention program's effectiveness among pupils with mathematical learning disabilities, among pupils at risk for mathematics failure, and among pupils without learning disabilities.

The four intervention programs examined in this study foster self regulated learning skills using explicit metacognitive guidance. They differ from each other on two effects: **the effect of timing of appearance of metacognitive prompts** (before or after the solution), and the **fading effect** (giving prompts according to the principles of the fading model, or giving fixed support). In both intervention programs that were conducted according to the fading model, the metacognitive prompts were gradually removed for the purpose of encouraging learners to internalize the prompts and to apply them on their own. Whereas in the intervention programs that operated without fading, prompts were given regularly during the intervention. The influence of these four programs was examined while comparing them to the program's effect on the control group, which was exposed to strategic instruction according to the new mathematics' learning program.

The sample included 373 fourth grade pupils from six schools in Haifa and the Northern District, who were divided randomly into five research groups, according to

the intervention programs. Intervention programs lasted about two months, and included 18 teaching hours. Research tools included tests for assessing numerical problem solving skills in three cognitive skill areas (TIMSS), questionnaires in the area of self regulation, and scales for self evaluation that were presented at three points in time: prior to intervention, immediately after, and three months after concluding the intervention. Similarly, after the intervention we documented the problem solving process verbally in real time, among 10% of the participants in each research group. These pupils were also interviewed about the intervention program. Additionally, three months post intervention, 10 of the 15 teachers participating in the program were interviewed, in order to examine the retainment effect and transfer of the program's principles across time.

**Following are the study's major findings:**

**The Fading Model:** a positive effect was found for employing the model, on the pupils' achievements when solving numerical reasoning problems and on the self-regulated learning process. This influence is notable compared to fixed support without fading, as well as compared to the control group. Furthermore, the use of **the fading model after solving** is particularly effective for solution of non-routine problems immediately after the intervention, for ability to transfer to a new subject, and for transferring across time in that area. Similarly, this intervention program was found to be effective in promoting high order thought processes, which was expressed by suggesting unique and creative methods for solution, using appropriate mathematical terminology. Furthermore, participants in this group used metacognitive statements (knowledge, control and judgment) more frequently than participants in the other groups while verbally solving a non-standard problem.

**Population groups:** the program with **regular support after the solution** was found to be the most effective for pupils with mathematical learning impairment, and actually contributed to improving their achievements in numerical reasoning problem solving and in mastery of metacognitive knowledge. Likewise, this program was also effective with pupils at risk for mathematics failure, both in terms of mathematical knowledge as well as in terms of self-regulated learning components (control of cognition, metacognitive judgments and motivation). By contrast, pupils without

learning disabilities were rewarded particularly by programs that integrated the **Fading Model**.

**This study contributes** to the theoretical, methodological and applied areas.

**In the theoretical realm**, the study is innovative regarding the use of a combination of self regulatory components, while nurturing metacognitive judgments among young pupils. The study's findings shed light on use of the Fading Model according to the timing effect when giving metacognitive prompts during numerical reasoning problem solving.

**Methodologically**, in the study's framework a broad variety of quantitative and qualitative measurement tools were employed (Mixed Methods). Furthermore, findings were validated through use of triangulation (questionnaires, interviews with participants and with teachers, and examining the process in real time) of research tools in order to cross check information from different sources. The present study thus clarifies the importance of using varied tools, especially qualitative processes, in real time.

**On the practical level**, teachers can use the intervention programs suggested in this study, while relating to the wide variety of pupils' skills in a heterogeneous class. These programs can create a basis for individual or group treatment according to differing levels of pupils' homogeneity in the class. Additionally, the programs can be fitted to age groups other than those examined in the study, and for transferring to various mathematical topics.